

Using GPS PPP To Test Relativity

Demetrios Matsakis

Civil GPS Service Interface Committee

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Special Theory of Relativity, 1905



- All observers are equivalent.
 - They all think they are stationary
- All observers will measure identical values for the speed of light.

Therefore,

- Nothing can go faster than light
- Honorable observers will disagree about
 - What is stationary
 - How long length is
 - What is simultaneous
 - i.e. what came first
 - How fast time moves
- But a freshman can easily calculate what different observers will think they observed

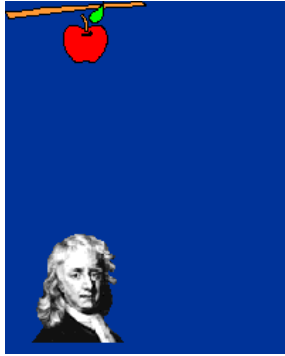
And the freshman says:

A stationary observer will think a moving observer's

- Yardstick shrinks
- Mass is greater
- TIME RUNS SLOW
 - Therefore, the clocks will run slow
 - 2nd order Doppler Effect



Questioning Gravity

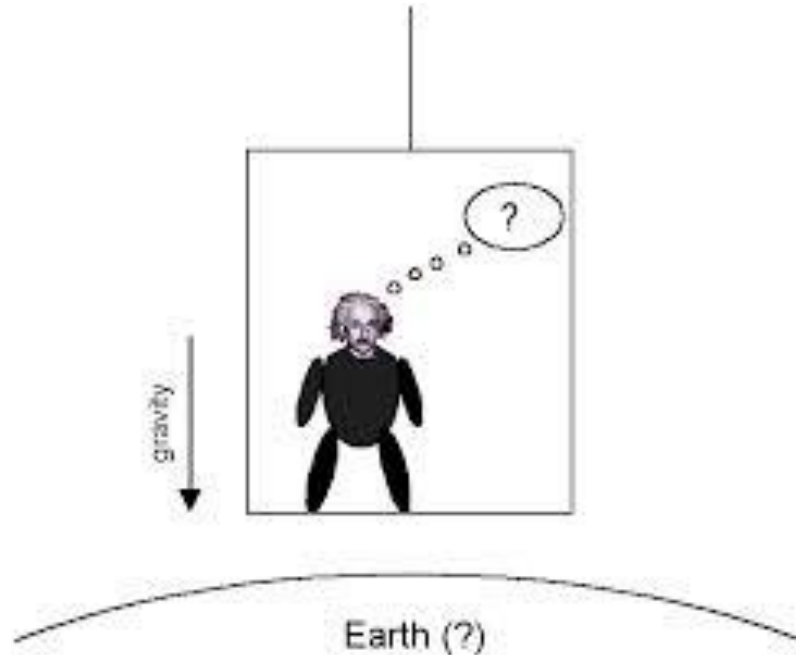


- How can gravity pass through empty space?
 - How can the Earth pull on an apple?
 - How can it pull on the Moon?
- Answer: the General Theory of Relativity

Newton from <http://csep10.phys.utk.edu/astr161/lect/history/newtongrav.html>

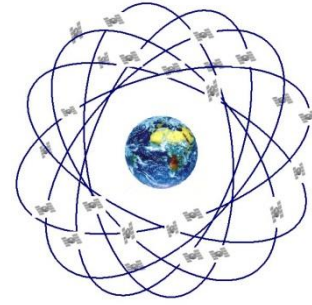
Moon from <https://en.wikipedia.org/wiki/Moon>

Principle of Equivalence, 1907

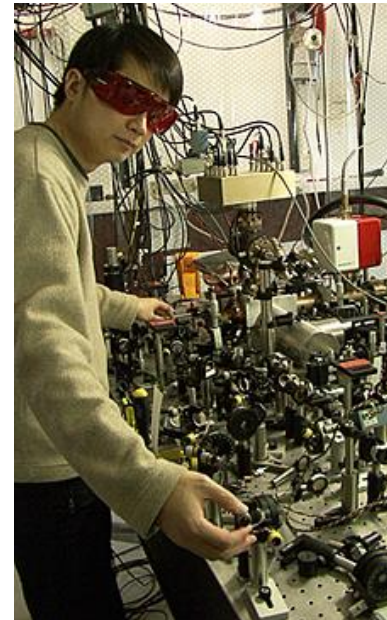


An observer cannot by local measurements, distinguish between an acceleration and a gravitational field.

The EEP explains the Gravitational Redshift



GPS, since 1978

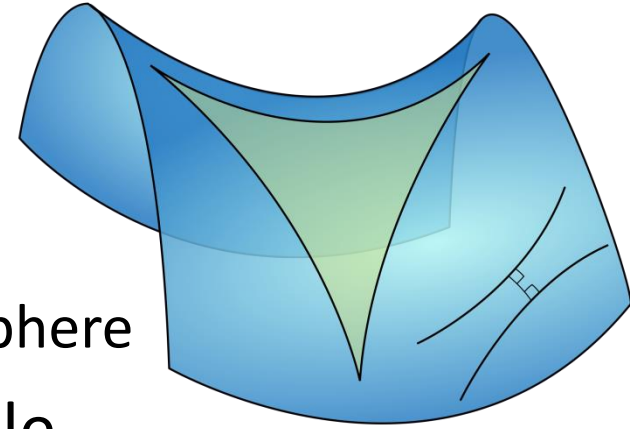


NIST, 2010



Harvard, 1959

Curved Space



Source: Wikipedia

- Euclidian geometry does not apply
 - Plane geometry does not work on a sphere
- Things move straightest way possible
 - Principle of Least Action



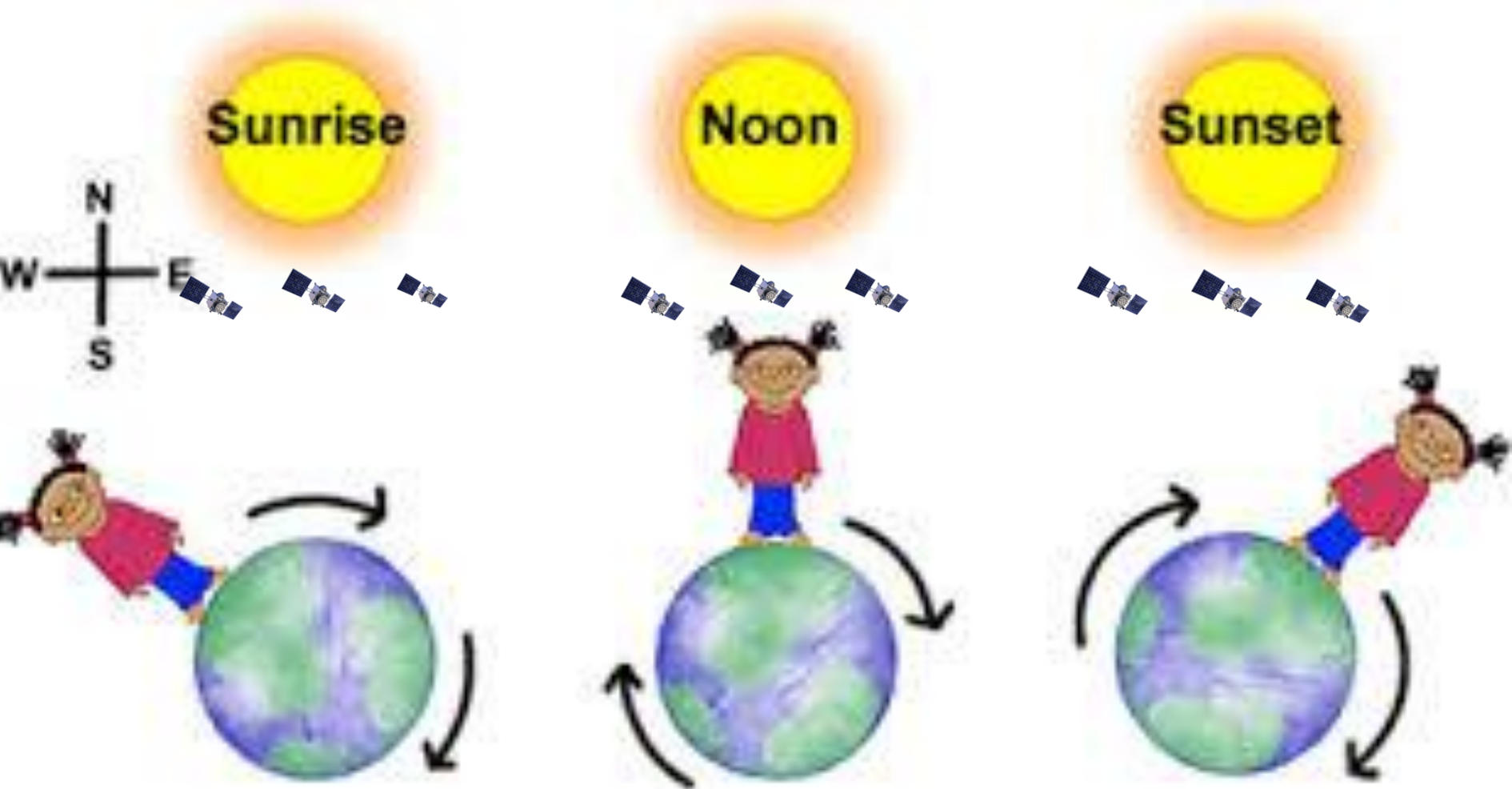
- Imagine a 2-dimensional ant walking around a sphere
 - She is going in a circle
 - She thinks gravity brought her back
 - You and Einstein realize that her space was curved.

Ant on globe from <http://cosmology.carnegiescience.edu/https@cosmology.carnegiescience.edu/sites/cosmology.ciw.edu/files/images/ball1.png>

Many theorists predict EEP violations
the question is where and how



We are closer to the Sun in the daytime, so its gravity is stronger.



Should the gravitational redshift from the Sun appear in GPS and ground clocks?

No, because all the clocks are, more or less, in free-fall about the Sun

- What about the Pound-Rebka & NIST experiments?
 - The equipment was tied to the stationary ground
 - An accelerating reference frame could replace gravity
 - But it saw the equipment as being accelerated by the ground
- Its different when everything moves together around the Sun

Cancellation of Gravitational & 2nd-Order Doppler shifts

Solar gravitational redshift:

$$\begin{aligned}\Delta f/f &= GM_{\text{sun}}/(c^2 D_{\text{sun}}) = (GM_{\text{sun}} * r_{\text{earth}})/(c^2 D_{\text{earth-sun}}^2) \\ &= (6.674 * 10^{-11}) * (1.988 * 10^{30}) * (6.378 * 10^6) / (3 * 10^{18} * 1.496 * 10^{11})^2 \text{ mks units} \\ &= 4.2 * 10^{-13} = 36 \text{ ns/day}\end{aligned}$$

Second Order Doppler about Sun:

$$\Delta f/f = -\frac{1}{2} v^2/c^2 = -GM_{\text{sun}}/(c^2 D_{\text{sun}}) = -36 \text{ ns/day}$$

Relativity of Simultaneity extends cancellation over orbit

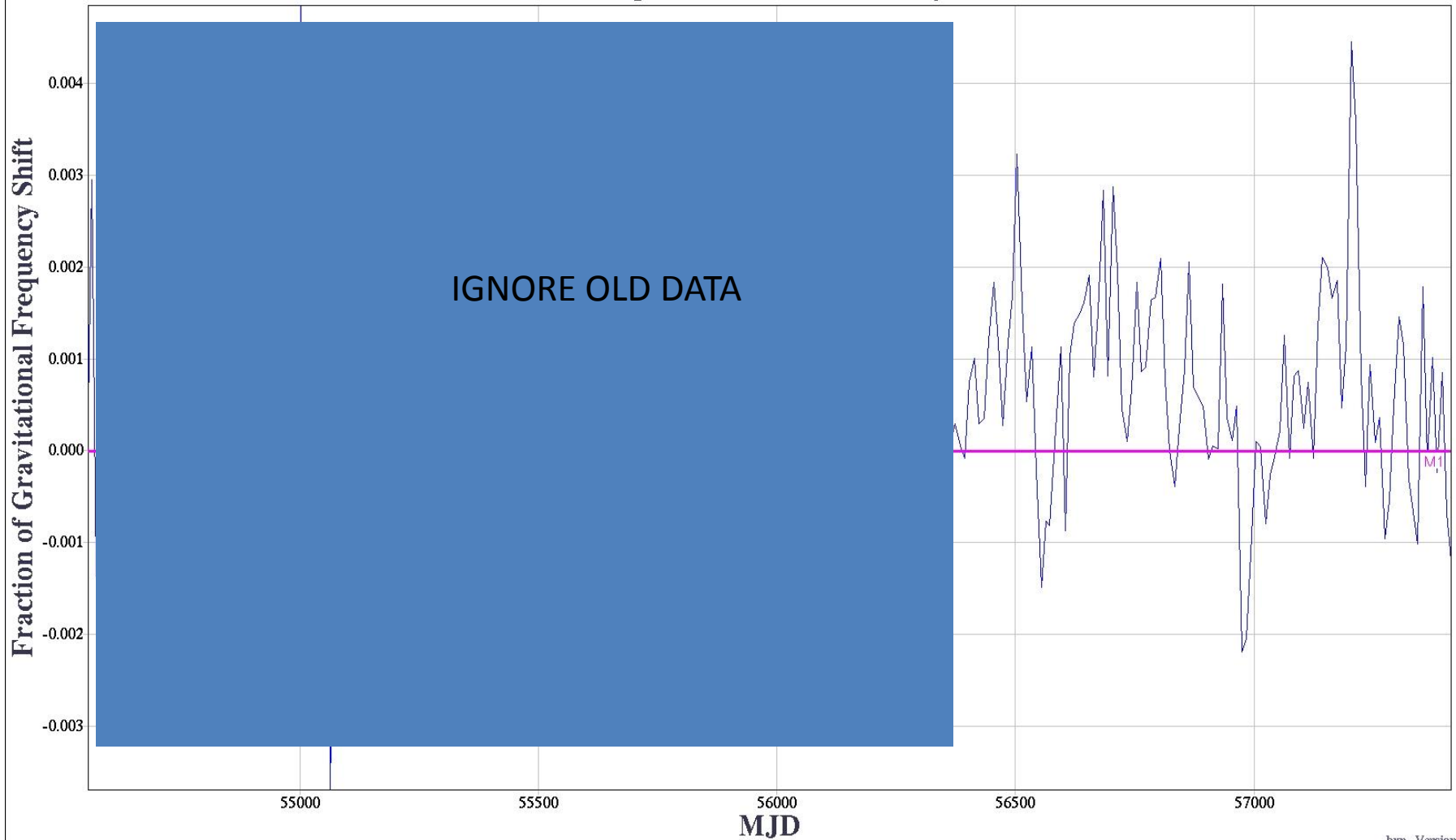
Ashby and Weiss (2013)

GPS Precise Point Positioning (PPP)

- Search for frequency shift as fraction of solar gravitational red shift
 - Which is supposed to be cancelled by the Doppler
- Monthly solutions computed by BIPM
- Ten labs used (initially)
 - 4 in Europe, 4 in Asia, 2 in USA
 - Data going back as far as 2008
- Sensitive to model errors in satellites visible at each site
 - EEP violation on satellites ~ 4 times larger than on surface clocks
 - That's a good thing for this experiment

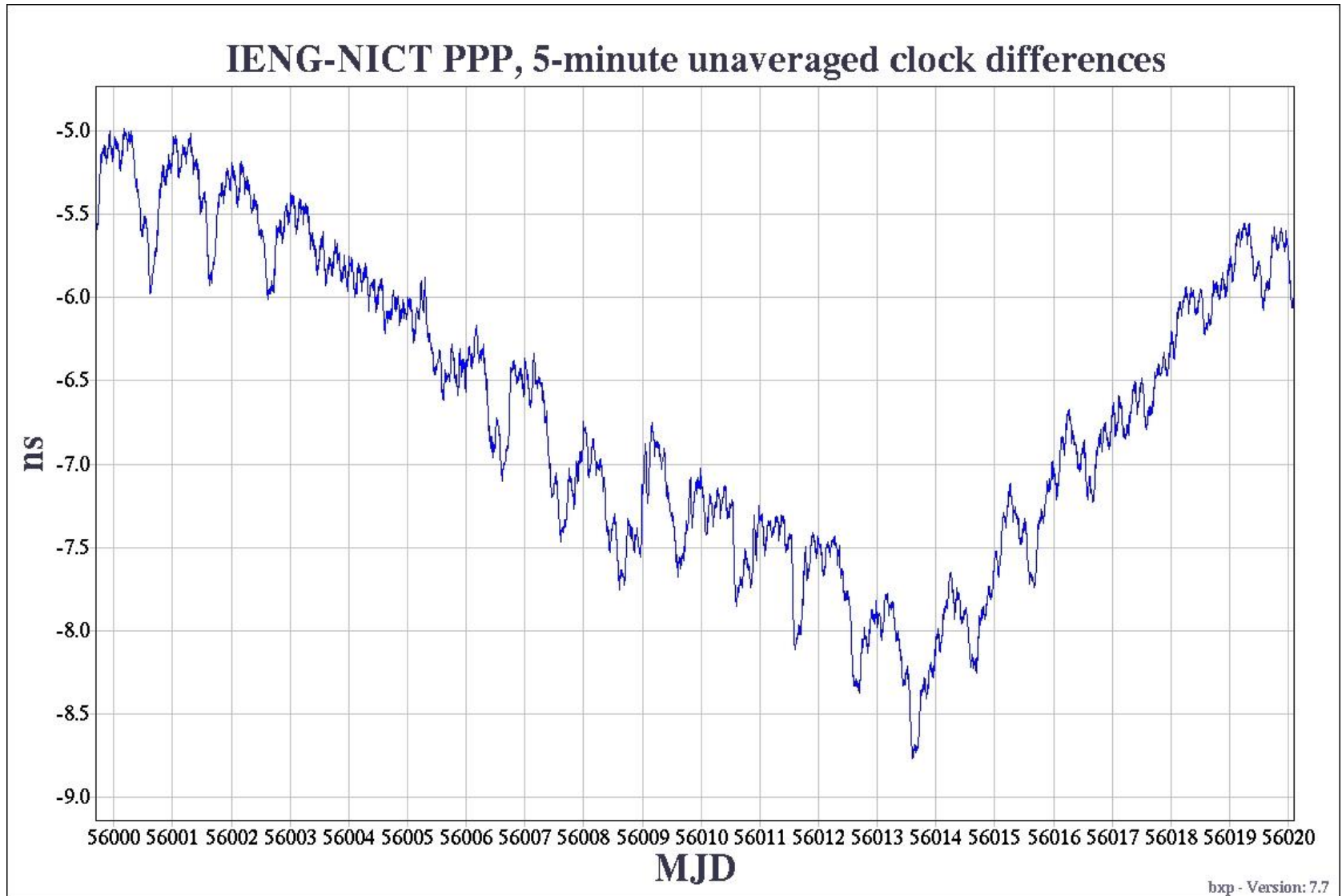
PPP RESULTS

EEP-Violating Parameter, PPP, 10-day smoothed



Intermittent Diurnals In PPP

(worst case, shown below, was excluded)



Bottom Lines

Technique	# Days	Covariance Source	EEP-violating Parameter	Statistical Uncertainty	Quadrature Parameter
TWSTT	1538	Data	$-2.2 \cdot 10^{-4}$	$\pm 2.4 \cdot 10^{-3}$	$5.0 \cdot 10^{-4}$
TWSTT	1538	Model	$1.4 \cdot 10^{-3}$	$\pm 2.4 \cdot 10^{-3}$	$5.5 \cdot 10^{-3}$
PPP	2865	Data	$7.4 \cdot 10^{-4}$	$\pm 1.1 \cdot 10^{-4}$	$4.4 \cdot 10^{-4}$
PPP	2865	Model	$7.9 \cdot 10^{-4}$	$\pm 8.9 \cdot 10^{-5}$	$7.8 \cdot 10^{-4}$
PPP	1217	Data	$6.8 \cdot 10^{-4}$	$\pm 1.1 \cdot 10^{-4}$	$2.8 \cdot 10^{-4}$
PPP	1217	Model	$6.1 \cdot 10^{-4}$	$\pm 9.2 \cdot 10^{-5}$	$4.4 \cdot 10^{-4}$



Systematics $< 10^{-3}$



Formal Error $\sim 10^{-4}$

Tests of Local Position Invariance

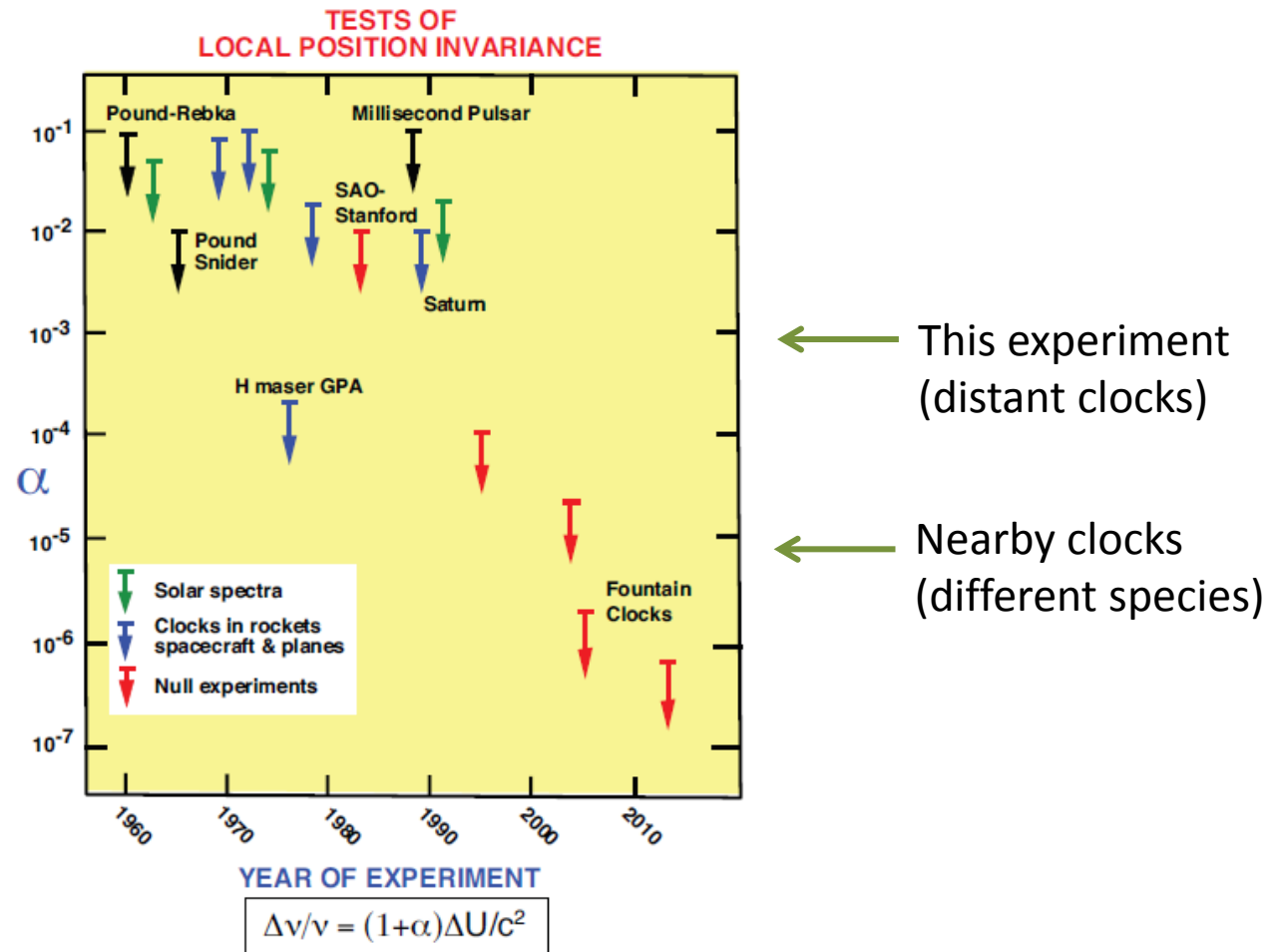


Figure 3: Selected tests of local position invariance via gravitational redshift experiments, showing bounds on α , which measures degree of deviation of redshift from the formula $\Delta\nu/\nu = \Delta U/c^2$. In null redshift experiments, the bound is on the difference in α between different kinds of clocks.

Future prospects involving Solar potential

- Everything getting better on Earth
 - PPP will be better
 - GPS => GNSS
 - Better hardware
 - Less environmental dependence, less multipath
 - Fiber-optics frequency transfer
- Circumsolar rocket probes
 - All they need is funding

Time's Up

